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SPACE VEHICLE
DESIGN CRITERIA
(STRUCTURES)

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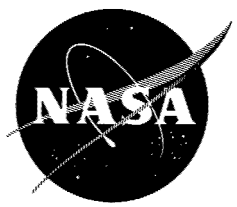
BUCKLING STRENGTH OF STRUCTURAL PLATES



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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

FOREWORD

NASA experience has indicated a need for uniform criteria for the design of space vehicles. Accordingly, criteria are being developed in the following areas of technology:

Environment

Structures

Guidance and Control

Chemical Propulsion

Individual components of this work will be issued as separate monographs as soon as they are completed. A list of all published monographs in this series can be found at the end of this document.

These monographs are to be regarded as *guides* to the formulation of design requirements and specifications by NASA Centers and project offices.

This monograph was prepared under the cognizance of the Langley Research Center. The Task Manager was A.L. Braslow. The author was R.H. Gallagher of Cornell University. A number of other individuals assisted in developing the material and reviewing the drafts. In particular, the significant contributions made by H.P. Adam and R.R. Meyer of McDonnell Douglas Corporation; M.F. Card of NASA Langley Research Center; W.J. Crichlow and R.E. Hubka of Lockheed Aircraft Corporation; S.B. Dong of the University of California, Los Angeles; N.F. Dow of General Electric Company; L.E. Hackman of North American Rockwell Corporation; R.N. Hadcock of Grumman Aerospace Corporation; and P. Seide of the University of Southern California are hereby acknowledged.

NASA plans to update this monograph when need is established. Comments and recommended changes in the technical content are invited and should be forwarded to the attention of the Design Criteria Office, Langley Research Center, Hampton, Virginia 23365.

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GUIDE TO THE USE OF THIS MONOGRAPH

The purpose of this monograph is to provide a uniform basis for design of flightworthy structure. It summarizes for use in space vehicle development the significant experience and knowledge accumulated in research, development, and operational programs to date. It can be used to improve consistency in design, efficiency of the design effort, and confidence in the structure. All monographs in this series employ the same basic format – three major sections preceded by a brief INTRODUCTION, Section 1, and complemented by a list of REFERENCES.

The STATE OF THE ART, Section 2, reviews and assesses current design practices and identifies important aspects of the present state of technology. Selected references are cited to supply supporting information. This section serves as a survey of the subject that provides background material and prepares a proper technological base for the CRITERIA and RECOMMENDED PRACTICES.

The CRITERIA, Section 3, state *what* rules, guides, or limitations must be imposed to ensure flightworthiness. The criteria can serve as a checklist for guiding a design or assessing its adequacy.

The RECOMMENDED PRACTICES, Section 4, state *how* to satisfy the criteria. Whenever possible, the best procedure is described; when this cannot be done, appropriate references are suggested. These practices, in conjunction with the criteria, provide guidance to the formulation of requirements for vehicle design and evaluation.

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BUCKLING STRENGTH OF STRUCTURAL PLATES

1. INTRODUCTION

A plate is a planar body whose thickness is small compared with its other dimensions. Flat or slightly curved plates are frequently used elements in space-vehicle structure. A plate structure, as defined herein, may be as simple as the flat web of a stiffener or as complex as an integrally stiffened plate supported by heavy frames and rings.

In the behavior of these plate structures under inplane compression and shear loads, a critical point exists where an infinitesimal increase in load can cause the plate surface to buckle. The load at this critical point defines the buckling strength of the plate. Increases in load beyond the load at the initiation of buckling increase the buckling deformations until collapse occurs. Thus, the load at collapse defines the postbuckling or crippling strength of the plate. The behavior of plate structures in this regard differs markedly from the behavior of columns and many thin curved shell structures for which the buckling load corresponds closely to the collapse load.

A fundamental problem in space-vehicle design is to size plate elements so that plate buckling will not induce detrimental deformations at any expected service load up to limit load (the maximum load expected in service), and that the design ultimate load (the limit load multiplied by the ultimate design factor) will not exceed the plate-crippling strength.

Buckling of a plate structure can cause an unacceptable degradation in the aerodynamic profile of a space vehicle. It can trigger general buckling of a larger structure because of a redistribution of loads; it can also affect the response to the structure sufficiently to cause failure from excessive displacement or fatigue or aeroelastic phenomena.

This monograph presents criteria and recommends practices for determining the buckling and crippling strengths of structural plates under various types of static loading, both mechanical and thermal. The document is concerned primarily with flat plates, but is also applicable to plates with shallow curvature. It explicitly treats the buckling and crippling of plates of unstiffened, corrugated, stringer-stiffened, waffle-stiffened, sandwich, and fiber-reinforced composite construction, but it does not treat beam-column effects, which are considered to be a stress-analysis problem.

The basic approach to determining the buckling strength of plate structures is through analytical or numerical solution of the linearized equations governing the transition from a flat form to a slightly buckled form. However, the analysis is different in some details for the various forms of plate construction, such as stringer-stiffened, waffle-stiffened, and sandwich panels. Crippling strength is generally determined semiempirically. A test program is usually required when analytical procedures or existing data are inadequate for determining buckling strength, as in panels with cutouts, local loadings, new forms of stiffening, uncertain support conditions, or new fastening methods.

Three other monographs in this series relate directly to the buckling of plate structures: those on the buckling of cylinders (ref. 1), of cones (ref. 2), and of doubly curved shells (ref. 3), which describe the instability analysis of complete structural systems of restricted geometric form. Other monographs are planned on design factors and on problems of estimating the thermal environment and dynamic response of structural panels.

2. STATE OF THE ART

For well-constructed flat plates, agreement between buckling theory and experiment is good enough to warrant determining buckling loads directly by analysis without empirical correlation factors. For curved plates, empirical corrections must be applied to analytically determined buckling loads; the crippling load is usually determined semiempirically. For unstiffened and stiffened plates of isotropic materials (plates whose material elastic properties are the same in all directions), the processes of determining buckling and crippling loads are well known and have been adequately summarized in references 4 to 11. For lightweight plates, such as those reinforced by sandwich construction and those of nonmetallic materials reinforced by fibers, orthotropic-plate-buckling theory has been applied (as used here, the term orthotropic denotes an analytical model in which the directional stiffness properties of reinforcement are accounted for). A number of buckling solutions for orthotropic plates can be found in references 12 to 14.

2.1 Analytical Methods for Stability Evaluation

The methods predominant in evaluating plate stability are discussed here under the categories of *classical* and *numerical*.

Classical methods, as defined here, represent plate-behavior parameters by mathematical functions that are continuous throughout the entire panel. The parameters used are generally the transverse displacements of the plate's middle surface. Classical

methods offer both closed-form and series solutions, which are derived by a direct attack on the governing differential equations or from the Rayleigh-Ritz or Galerkin procedures for manipulating these equations. References 4 to 6 can be consulted for descriptions of these classical-solution procedures.

Closed-form solutions fully determine panel-behavior parameters under all equilibrium, continuity, and boundary conditions. In practice, however, the cases amenable to closed-form solutions are limited, and methods leading to series-form solutions are more generally applicable. The form of the series for a particular problem must be chosen so it can be truncated at a finite number of terms while approaching exact satisfaction of the governing equations in the limit.

In theory, classical methods can deal with all the phenomena of flat-plate stability, including inelastic buckling and postbuckling (crippling) strength. Providing the proper treatment of these phenomena by classical methods is a matter of being able to represent parameters in the basic equilibrium, constitutive, and strain-displacement relationships. This representation can best be accomplished for rectangular plates with simple boundary conditions. Irregular geometry or nonuniformly distributed applied loads or temperatures are extremely difficult to represent in parametric form, and the flat-plate stability problem can therefore be solved by classical methods in only a few special cases when such conditions exist.

By contrast, *numerical* methods characterize the behavior of a structure at points or within regions of the structure and result in large-order systems of equations whose coefficients are numerically evaluated functions of the material, geometry, and applied-load parameters at these points or regions. As a group, these methods furnish wide latitude in the treatment of nonuniformly distributed values of the design parameters and nonlinear behavior, but solutions cannot be represented parametrically as readily as they can with classical solutions. The finite-difference and finite-element procedures are the foremost numerical methods.

The finite-difference method (ref. 15) represents the traditional approach to numerical analysis; its well-established principles lead to definition of the correct solution and assure convergence to it as the gridwork of points is refined. Moreover, there is an extensive body of experience in applying this method to practical problems. The resulting algebraic equations are relatively simple, permitting the use of highly efficient solution techniques. On the other hand, the finite-difference method in its usual form can evaluate only that stability behavior which can be described by one or more differential equations, and it is difficult to apply if it must deal with irregular gridworks or boundaries.

At present, the finite-element method is well established as a procedure for the static and dynamic analysis of complex aerospace structures under stable conditions (refs. 16 to 18). Application of this method to instability problems is a more recent development (refs. 19 to 21), and verification of the techniques for instability problems for plates is limited. If convergence of the finite-element model has not been well established when a complex plate structure is analyzed, the solution is usually verified by experiment. Progress in finite-element instability analysis is surveyed in reference 22.

Numerical methods and many applications of the classical methods based upon series representations require the computation of the buckling-load intensity and the associated buckling-mode shapes from the coefficients of large-order systems of linear equations. The mathematical description of buckling-load intensity is the eigenvalue, and the mathematical description of mode shape is the eigenvector. Reference 23 is a detailed treatise on the numerous procedures available for the computation of eigenvalues and eigenvectors. There is a great need to improve computer programs to the point where they can solve large-order eigenvalue problems efficiently and accurately.

2.2 Stability Evaluation of Plates of Typical Construction Forms

A relatively few construction forms of plates predominate in space vehicles; these are largely unstiffened, corrugated, stringer-stiffened, waffle-stiffened, sandwich, and fiber-reinforced composite plates. In practice, some combination of analytical and empirical techniques is used to evaluate the buckling and crippling behavior of each class of plate construction, as discussed in the following sections.

2.2.1 Unstiffened

Formulas for buckling stress for flat, rectangular, unstiffened plates have been determined for many support and load conditions, and have been substantiated by test. Buckling stresses for geometrically irregular plates, especially those containing cutouts, are not easily determined, however, and evaluation of stresses in such plates requires use of numerical methods.

If the buckling stress of an unstiffened plate exceeds the material's elastic limit (i.e., is in the plastic range of the material), buckling strength can be calculated in essentially the same manner as the elastic-buckling strength of orthotropic plates, with coefficients for terms in the governing equations drawn from functions of the moduli in the nonlinear region of the material's stress-strain diagram (ref. 24).

Formulas derived from this approach to evaluating plastic buckling for simple plate geometries and load conditions can be expressed as the familiar elastic-buckling equation multiplied by a plasticity-reduction factor, η_p.

Two major approaches have been taken in determining η_p analytically, one based on the flow theory (ref. 25) and the other on the more simplified deformation theory (refs. 26 and 27). Incremental or flow theories work with incremental stress-strain laws in which increments of plastic strain are expressed in terms of the stress. Analysis of the structure involves an integration of the plastic strains over the complete history of loading. Deformation theory of plasticity is defined by a stress-strain law in which the total strain components are related to the current stress and are therefore assumed to be independent of the history of loading.

Experimental evidence (e.g., ref. 28) has confirmed the suitability of deformation theory in flat-plate inelastic-buckling analysis. Vol'mir (ref. 6) has suggested that without a linear relationship between applied load and stress up to the level of the load that causes buckling, the deformation theory in plastic buckling may be limited for plates with significant initial imperfections. No experimental studies which could confirm or deny this suggestion have yet been reported.

Although experimental data in substantiation of plastic-buckling theory pertain largely to uniaxial compression, the analogy drawn in reference 24 to the applicability of orthotropic-plate theory has been used successfully for biaxial compression. Reference 29 describes the basic theoretical relationships for more general stress states, including shear, but does not give detailed solutions for such cases.

Both the postbuckling stiffness and the crippling strength of isotropic flat plates have been studied extensively. The new analytical procedures for treatment of nonlinear phenomena of postbuckling behavior for all forms of structures under compression were motivated by Koiter (ref. 30) and considerably developed by other theorists (refs. 31 and 32).

Flat-plate postbuckling behavior under axial compression has been described in references 33 and 34. Reference 35 extends this work and develops the design charts; it also contains the computer program listing from which the plotted data of the design charts were obtained. The shear stiffness of a plate buckled in axial compression is given in references 36 and 37; the special case of postbuckling stiffness under shear loading is treated in references 36 and 38.

To date, most theoretical approaches to evaluating crippling strength (e.g., ref. 39) assume that the flatness of the plate is ideal and that the shape of the postbuckling displacements can be defined on the basis of the initial buckling mode. Recent studies

(refs. 37, 40, and 41) have demonstrated the inadequacy of these assumptions at the onset of buckling failure, a point reached when yielding first occurs in some highly deformed region of the middle surface.

The complexity of a nonlinear analysis that would account for uncertainties in evaluating crippling strength has led to the adoption of various simplified procedures. A common approach is based upon the concept of effective width; that is, the assumption that the entire collapse load is taken by uniformly stressed widths of material adjacent to the supports. The stress for crippling is assumed to be the material's yield stress. The many formulas proposed for the determination of so-called "effective" widths are discussed and compared in reference 42, which demonstrates that no single formula is valid for all types of loading and support conditions.

An alternative approach to crippling-strength analysis which has been correlated with extensive test data is advocated in reference 43. This approach relies upon a material property, the "crippling-strength parameter," defined as a function of the secant modulus of elasticity and the compressive-yield strength. Thus, crippling-strength-parameter data obtained from tests of one material can be used in calculation of the crippling strength of a plate of different material. Although intended to cope with elevated-temperature design problems, the approach is suitable for determining the crippling strength of plates in structures under other conditions.

Theory regarding the determination of crippling strength of plates that buckle inelastically is essentially underdeveloped. However, one indication from experiment is that only small differences exist between inelastic buckling- and crippling-load levels.

Thermal buckling, initial imperfections, and plate curvature deserve special comment. Thermal buckling occurs when constraint against thermal expansion in the middle surface of a plate produces enough compressive stresses to cause buckling. The constraint may be imposed by the plate-edge members or may be within the plate as a result of the variation of thermal strain from point to point.

Since thermal-stress conditions often result in nonuniformly distributed compressive stress, it is difficult to evaluate thermal buckling. Furthermore, it may be necessary to account for the thermal effects on material properties in this evaluation. Thus, relatively few approaches for defining thermal buckling have been published; these are cited in the annotated bibliographies of references 44 and 45, and surveyed in reference 46. In general, these references deal merely with elastic buckling under nonuniform stress and are not uniquely associated with phenomena resulting from elevated-temperature conditions. Moreover, empirical data on postbuckling behavior and crippling strength under thermal stress are scarce. The problem is complicated because the stress system causing buckling (which arises as a result of the constraint of

midplane thermal expansion in the unbuckled state) is modified and, in general, alleviated by the transverse (flexural) displacements in the postbuckling phase. The governing nonlinear equations for this behavior have been presented and solved for specific cases in references 47 and 48, but these solutions are limited in application. A simple, semiempirical approach to determining the onset of permanent buckles in the presence of thermal stress is proposed in references 49 and 50.

Initial imperfections (i.e., those deviations from flatness due to fabricational inaccuracy which all plates possess to some degree) cannot be described adequately by simple functions. It is nevertheless possible to draw general conclusions concerning the influence of such imperfections on buckling through a criterion for the relationship between applied load and deformation at the instant of buckling embodied in Koiter's previously cited approach to nonlinear analysis (ref. 30). Applying this criterion, reference 32 demonstrates theoretically that initial imperfections do not significantly influence the buckling loads of flat plates. This is in accord with long-standing experimental evidence.

When discussing the influence of curvature on plate stability, it is useful to distinguish between plates with shallow deviations and those with deep deviations from flatness. Theoretically, when considered as a small deflection, curvature has the effect of increasing the initial buckling level beyond that of a corresponding flat plate. From an empirical point of view, however, it is evident that the imperfection-sensitivity of curved shells grows with the increase of curvature and reduces the actual buckling level below that predicted by a linear theory which includes curvature. Early design data (e.g., refs. 51 and 52) therefore represent a correlation of theory and experiment. More recently, reference 53 has shown that solutions of the shallow-shell differential equations of reference 54 for the limiting case of zero curvature correspond with flat-plate solutions only when special types of edge conditions exist. No parametric representation of the results of these solutions is feasible because of the complexity of the problem and the many relevant parameters. The survey of the work given in reference 53 attests to the paucity of prior experimental and theoretical efforts.

Reference 55 conjectures that the behavior of a buckled singly curved plate asymptotically approaches that of a buckled flat plate. In the absence of experimental data on the crippling strength of singly curved plates, this conjecture leads to the use of flat-plate theory for crippling analysis. There is no experimental evidence concerning the crippling strength of doubly curved plates.

2.2.2 Corrugated

Corrugated plates were used to form the surfaces of early metal aircraft to meet a design requirement which called for withstanding only shear loads, although these

plates have significant uniaxial buckling strength. At present, design interest in corrugated plates has been renewed because of their ability to minimize thermal stress. A corrugated plate with a single face sheet is treated as a stringer-stiffened plate (Sec. 2.2.3); with two face sheets, it is treated as a sandwich plate (Sec. 2.2.5).

Analytical methods for calculating initial corrugated-plate buckling have been based almost exclusively upon conventional orthotropic-plate theory (refs. 56 to 58). Such calculation is often in poor agreement with the test data because of the great flexibility existing in the plate in planes perpendicular to the corrugations, the practice of attaching edges at discrete points in real structures (i.e., riveting), and inconsistencies in the procedures for determining the effective twisting stiffness for use in equivalent orthotropic-plate theory.

The effect of flexibility in planes perpendicular to the corrugations is analogous to the effect of transverse shear deformation. Reference 59 accounts for this effect by use of shell theory, but the complexity of the solution limits application of this approach to plates with a shallow sinusoidal form of corrugation. Recent studies (refs. 60 and 61) indicate that the theory of thin-walled beams with deformable cross section promises to be applicable for more realistic evaluation of corrugated structures.

References 56, 62, and 63 consider the intermittent attachment at corrugated-plate edges in dealing with initial buckling under pure shear. In contradiction to experimental data and St. Venant's principle, these references employ assumptions implying that edge effects propagate throughout the plate. References 59 and 60 also report studies directed toward realistic intermittent-edge attachment.

For corrugated plates, analytical representation of local buckling, postbuckling strength (ref. 59), and plastic buckling is in its infancy. Available information demonstrates significant discrepancies between buckling theory and test data for corrugated plates.

2.2.3 Stringer Stiffened

As shown in figure 1, a stringer-stiffened plate consists of a sheet with attached stiffening members along the direction in which the predominantly uniaxial loads act. Stringers, either integral with the sheet or attached to the sheet by rivets, spot welds, or adhesives, efficiently enhance panel stiffness. A large number of stringer cross-sectional shapes are possible; however, the economics of fabrication favor the forms illustrated in figure 1.

Early methods for stiffened-panel buckling analysis, as summarized in references 10 and 11, treated initial buckling as synonymous with the instability of the sheet

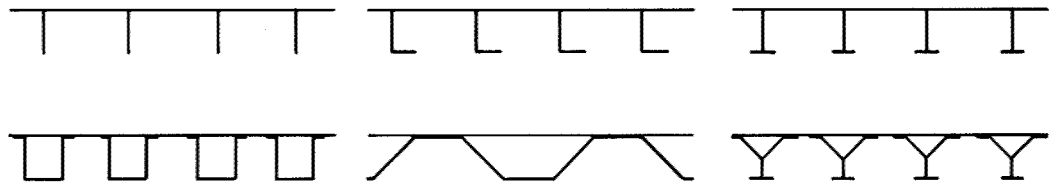


Figure 1. – Types of stiffeners used in stringer-stiffened plate construction.

between stringers, and computed crippling strength by analysis of a stringer and a portion of the adjacent sheet in isolation. This model adequately represented the behavior of structures within the design requirements of the time and was substantiated by extensive test data.

Requirements for retaining a smooth structural profile up to the level of the initial buckling load (required today for many space-vehicle designs) and the increased emphasis on structural efficiency under failure loads motivated the development of different analytical models for stringer-stiffened plates. Both the coupled action of the sheet and stringers (ref. 64) and the stiffener eccentricity (ref. 65) are therefore recognized in current formulations for determination of initial buckling loads. For example, a general method exploiting the capabilities of the computer was recently devised for analyses of the coupled modes of buckling for stiffened-plate construction (ref. 66); an extension of this method has been developed for composite-reinforced stiffened plates (ref. 67). General methods using the finite-element approach have also been developed. Similar advances have been made in formulations for determining the crippling strength of stiffened plates (refs. 68 to 70).

Excellent summaries of practice in the analysis of stiffened plates up through the mid-1950s are presented in references 10 and 11. Since 1960, less attention has been given stiffened-plate instability analyses. This is due in part to satisfaction with existing formulations, reduced emphasis on this form of construction, and anticipation that emerging computer procedures will improve the efficiency and accuracy of currently available analytical methods.

2.2.4 Waffle Stiffened

A waffle-stiffened plate consists of a grid of stiffeners integral with each other and with the skin of the plate (fig. 2), and derives its structural efficiency from geometric uniformity and the absence of attachments.

Overall analysis of the buckling of waffle-stiffened plates involves two procedures: (1) the transformation of the basic geometric proportions into the coefficients, or

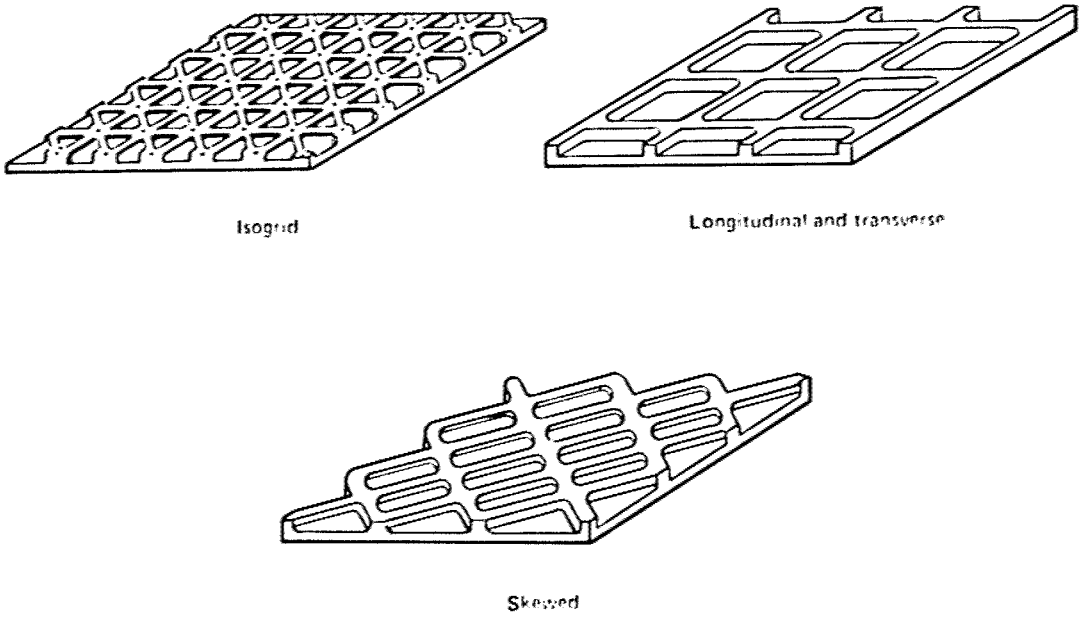


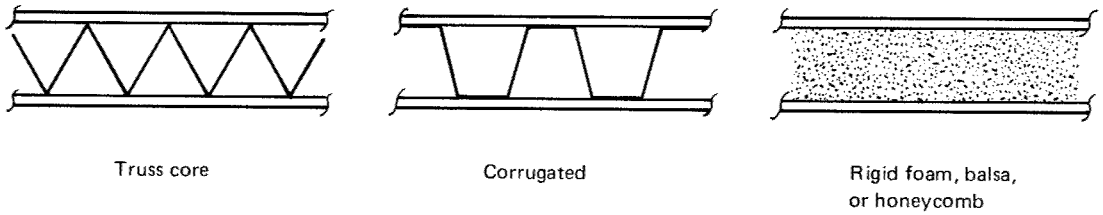
Figure 2. – Types of waffle stiffening.

elastic constants, relating the stress resultants to the strain parameters; and (2) the solution of the governing buckling equations. Published studies have dealt principally with the former procedure (refs. 71 and 72); more recent studies (ref. 73) have included the effects of eccentric stiffening. Since waffle-stiffened structures have been mainly in the shape of monocoque cylinders, the buckling of waffle-stiffened flat plates has not been directly evaluated. There is little experimental information on the plastic-buckling or postbuckling performance of waffle-stiffened plates.

2.2.5 Sandwich

Sandwich plates are formed by bonding two face sheets to a core (fig. 3). The face sheets generally provide all of the plate's flexural rigidity; the core separates the faces and transmits shear. Sandwich plates may fail in either an overall mode of buckling similar to the buckling of unstiffened plates or in a variety of local modes, as illustrated in figure 4.

Many theories of varying degrees of complexity have been formulated to analyze the overall mode (fig. 4a) of buckling of sandwich plates (e.g., ref. 74). These theories differ in their approaches to idealizing the core and face deformation. Theories must usually account for transverse-shear deformation of the core. Unified treatments from which both the overall and local modes of buckling can be calculated have also been developed (e.g., ref. 75). In most practical situations, the results obtained for overall buckling by application of either particular or unified theories are essentially the same.



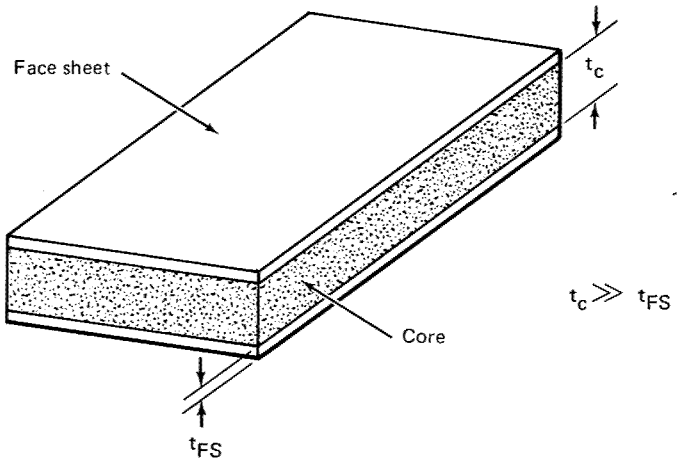


Figure 3. – Sandwich-plate construction.

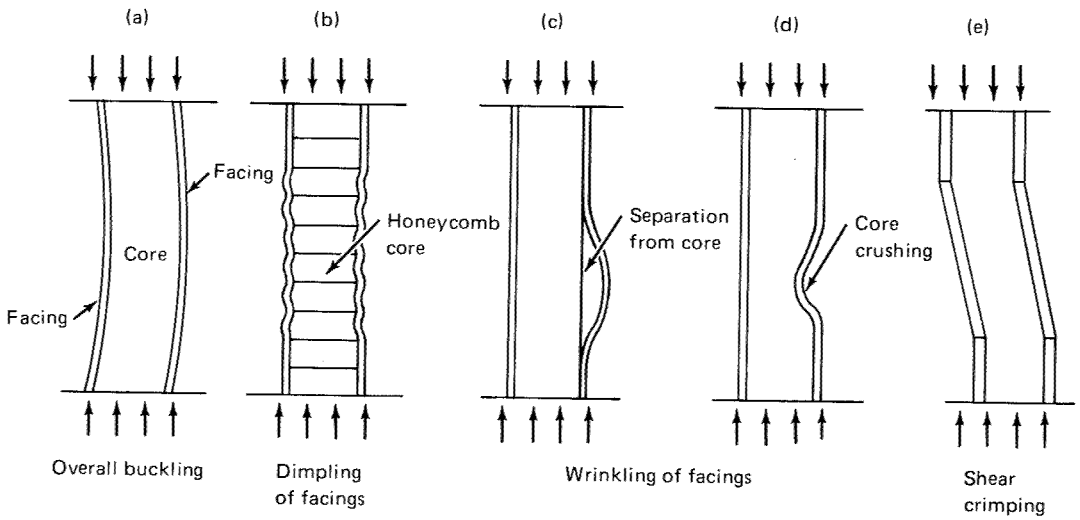


Figure 4. – Typical failure modes of sandwich plates.

Moreover, unified approaches to the description of local buckling phenomena have thus far demonstrated no special advantage over treatments based upon selective analytical models because empirical considerations predominate in the description of such phenomena.

Plastic buckling of sandwich plates has been treated either (1) by replacement of the elastic modulus of the faces with properties of the stress-strain law in the plastic range (ref. 75); or (2) to account for plasticity, by correction of the flexural stiffness in a plate-buckling theory that includes transverse-shear deformation. All theories assume elastic behavior of the core and only isotropic faces have been considered in these descriptions, so experimental confirmation is limited. Moreover, there is no verification that the more complicated methods yield more reliable results than the simplest approximations. Theoretical understanding and experimental evidence on postbuckling strength are even more limited than on plastic buckling.

The principal modes of local buckling for homogeneous and cellular-core sandwich plates are dimpling (fig. 4b) and face wrinkling (figs. 4c and 4d); local buckling in this form of construction, as in truss-core and corrugated-core sandwich construction, is synonymous with failure. Shear crimping (fig. 4e) is not a local but a general instability mode in which the buckle wavelength is very short due to a low value of the transverse-shear modulus of the core. Shear crimping is accurately characterized by an easily derived theoretical formulation (ref. 76).

In the face-wrinkling mode of local failure, the sandwich facing buckles as a plate on an elastic foundation. Intracellular buckling (also called dimpling or monocell buckling) is the buckling of the face into the spaces found in cellular- and corrugated-core constructions. Since the occurrence of wrinkling strongly depends upon difficult-to-define plate characteristics such as waviness of the face, the elasticity and strength of the core, and the bond between the face and core, theoretical descriptions (e.g., refs. 77 and 78) have generally proven unsatisfactory, and useful formulas are based upon experimental data. Monocell-buckling formulas (refs. 79 and 80) are of semiempirical form and have been adequately verified by test.

Evaluation of sandwich plates with corrugated cores requires that core properties be represented orthotropically and assumes infinite transverse-shear stiffness in planes parallel to the corrugations. Corrugated-core sandwich plates have significant postbuckling strength; in common with other forms of sandwich construction, corrugated-core plate is, however, subject to failure in local buckling and inelastic buckling modes. Reliable methods are available for the analysis of each of these modes of behavior (ref. 81).

The historical development of sandwich-plate analysis is critically reviewed in reference 82; detailed examination of related theory is presented in references 81 and 83. A comprehensive review and compilation of analysis data, which serve as a detailed state-of-the-art assessment, are contained in reference 84. References 81 to 84 also report on isolated published studies of the effects of such factors as thermal stress, panel curvature, and initial imperfections on sandwich-plate buckling.

2.2.6 Fiber Reinforced

Fiber-reinforced composite plates are made of high-strength, high-stiffness fibers immersed in a matrix, or filler material, of lower strength and stiffness. The conventional fiber-reinforced composite plate consists of several plies, with long fibers or filaments oriented in a single direction in each ply. Reference 14 is an authoritative compendium of detailed analysis and design information on fiber-reinforced composite plates. It is continually updated as a result of on-going research.

During the early period of the development of fiber-reinforced composite structures, it was hoped that designs could be analyzed solely on the basis of the properties of the constituent materials (i.e., the fibers and the matrix). It has become apparent, however, that the most reliable fundamental material-property data are obtained from tests of the individual ply or lamina. The stiffness properties of the composite plates in turn are usually determined by integration of the individual ply properties with the formulas given in reference 14.

Basic theoretical procedures for calculating composite-plate stiffness from the properties of the individual laminas (ref. 14) depend on the coupling between inplane-membrane and plate-flexural behavior. The governing equations for evaluating fiber-reinforced composite-plate buckling are those of an orthotropic plate if no coupling exists (e.g., when the laminas are properly arranged about the plate's middle surface) or is insignificant enough to be eliminated from consideration (e.g., when the plate is composed of a large number of cross-plied elements). References 12 and 13 present design data for orthotropic plates applicable to fiber-reinforced composite plates with negligible coupling. Under certain conditions, the buckling analysis of fiber-reinforced composite plates may even be reduced to the form used for evaluation of isotropic plates. Experiments and comparative theoretical studies of fiber-reinforced composite-plate buckling (refs. 85 and 86) have generally been based on membrane-flexure coupling. These studies have confirmed the validity of using approximate procedures when the coupling is weak.

As yet, there is no complete understanding of all local failure modes in composite plates. Local yielding prior to buckling is possible, and may invalidate an elastic-buckling computation. A special criterion necessary for determining yielding in fiber composites has been formulated by Tsai, and is discussed in reference 87. The criterion requires careful engineering judgment in determining allowable yield-stress input, however, and has been exercised with only limited success for shear and compression loadings. Other criteria are discussed in reference 88. Experiments have shown that the delamination mode of local failure represents an important limitation on the post-buckling strength of fiber-reinforced composite plates (ref. 89). Because of the many geometric and material factors involved, extensive work is being undertaken to develop

suitable descriptions of the local failure modes and the postbuckling and inelastic strengths of fiber-reinforced composite plates (e.g., ref. 90).

3. CRITERIA

The buckling and crippling strengths of the plates of space-vehicle structures shall be determined with sufficient accuracy to ensure that (1) any buckling deformation resulting from any expected service load up to limit loads will not impair the function of the plate itself or any other system, nor produce any unanticipated changes in stiffness or load distribution that are not accounted for in the overall structure; and (2) plate crippling will not occur under application of design ultimate loads. Realistic representations of the applied load and temperature conditions, the material properties, and the structural geometry, details, and support conditions shall be incorporated in appropriate analytical and test models. Elastic or inelastic analyses of plate buckling and crippling strengths shall be conducted, as appropriate, and shall be substantiated by experimental data, when necessary.

3.1 Analysis Input

The values for applied stresses employed in determining plate buckling and crippling strengths shall represent conservative estimates of stress fields resulting from the combination of mechanical, thermal, and residual stress loading under all expected service conditions. Any significant dependence of the mechanical and physical properties of the material upon environmental conditions shall be accounted for. The plate stresses shall be derived from stress analyses of overall structure, and the individual plates shall be realistically represented in the overall structure model used in these analyses.

3.2 Analysis Procedures

The buckling and crippling strengths of structural plates shall be determined by calculation. Plate-support conditions shall be defined by realistic determinations of elastic fixity or, where this is not feasible, by conservative assumptions.

The following, as a minimum, shall be taken into account in the analysis when applicable:

- Significant variation of the material's constitutive relationships with geometry, time, and load level
- Joints, attachments, and cutouts

- Local failure modes, such as:
  (a) Interrivet buckling, torsional rolling, and stringer crippling of stringer-stiffened plates

  (b) Face wrinkling, face dimpling, shear crippling, and general buckling of sandwich plates

  (c) Yielding or delamination of composite laminates prior to buckling

- Combined loading effects

3.3 Tests

3.3.1 Materials

When adequate data on material properties are not available, they shall be obtained from standard material-property tests.

3.3.2 Plates

When the determination of plate buckling and crippling strengths by calculation cannot be shown to be adequate, representative tests shall be conducted to confirm the design. Pretest documentation shall demonstrate that the test's boundary conditions are valid and that the test loads, specimen, fixtures, instrumentation, and procedures are appropriate.

4. RECOMMENDED PRACTICES

Analysis to determine the buckling and crippling strengths of structural plates begins preferably with the definition of the loads applied to the plate, the geometric representation of the plate, and the values for the material properties of the plate. The analyst should then specify and solve the analytical model, review the problem for critical design details, and define the need for supporting test programs. It is usually necessary to repeat this process several times because of changes and refinement in the total vehicle design.

4.1 Analysis Input

The definition of loads applied to the plate should be based upon proper analytical modeling of the entire vehicle structure or of the major component of which the plate

forms a part. Such features as cutouts, concentrated loads, and shear-deformation effects in the skin of stiffened-shell construction should be represented in the model. The elastic-edge fixity of individual plates should be determined realistically from the results of a numerical-methods analysis of overall structure, from a detailed study of the design of individual elements, or from tests.

The plate analyst and the personnel responsible for the analysis of the complete space-vehicle structure should coordinate closely the determination of individual plate loads. With few exceptions, proper analysis of the complete structure depends upon the relative elastic characteristics of the components of the system, characteristics that are in part a function of the dimensions of the component plates. The plate analyst should therefore make sure that any changes in plate proportions motivated by his analysis are reflected in values for elastic characteristics used in the analysis of the complete vehicle structure. To ensure consistency of plate and vehicle design, the correlation of plate analysis with vehicle analysis should be repeated until plate loadings in successive iterations agree to within 5 percent or less.

Thermal stresses should be determined by realistic assessment of temperature changes expected from the environments, and the constraints on individual plates; residual stresses should be determined by realistic assessment of fabricational conditions (e.g., ref. 91). Since the effect of postbuckling behavior on these stresses is uncertain, stress intensity should be assumed to be unchanged in calculating crippling strength unless empirical data allow a different assumption. Flight conditions should be realistically represented when externally applied loads are combined with loads due to thermal stresses. Effects of temperature on material properties should also be considered in evaluating combined loads.

4.2 Analysis Procedures

4.2.1 Geometric Representation

Basic geometric data on plate size, curvature, and support conditions are required for the definition of an analytical model of a plate. Overall geometric data should normally be obtained from layout drawings of the space vehicle; occasionally, geometric data must be experimentally determined.

Preferably, support conditions should be determined from analyses of the major component of which the plate forms a part, or of the complete space-vehicle structure, as recommended in Section 4.1. Otherwise, support conditions should be determined from elementary analyses of the adjacent plates and stiffening members; in the absence of such analyses, simple support conditions should be assumed for edges possessing essentially complete constraint against transverse displacement.

4.2.2 Material and Structural Properties

Values for the properties of plate materials should be obtained from MIL-HDBK 5A (ref. 92) for metals, and values for the properties of materials and plate rigidities of sandwich-plate construction should be obtained from reference 93. Reference 14 should be consulted for the basic properties of fiber-composite materials. Values for a wide range of elevated-temperature and cryogenic properties for metallic materials are given in references 94 and 95.

In determining plate stiffness, it should be kept in mind that fiber-reinforced composite and waffle-stiffened plates present special problems. Stability analyses of these forms of construction are generally conducted with orthotropic-plate theory. Coupling between the flexural and membrane stiffnesses exists in an orthotropic plate, but the consequences of this coupling associated with direct stress are generally more important in evaluating buckling than are the consequences of coupling of stiffnesses associated with shear stress.

The coupling between membrane and flexural stiffnesses is usually weak for laminated-fiber composite plates, and the stability of these plates can be determined with sufficient accuracy by reducing the stiffness relationships to the form of orthotropic-plate bending properties with the use of the approximation described in references 85, 86, and 96. The inadequacy of existing procedures for computing individual layer-stiffness coefficients from the filament and matrix properties makes it necessary that these coefficients be derived from tests which establish the biaxial extensional stiffnesses and the shear stiffness in the plane of each individual layer. Stiffness properties should then be calculated for the complete plate by using the integration procedures given in Section 3 of reference 14.

Plate-stiffness coefficients for waffle-stiffened construction should be computed from formulas presented in reference 71. Although an approximate reduction of this form of stiffness representation to orthotropic-plate flexure is feasible by using the approach cited for fiber-composite plates (refs. 85, 86, and 96), simpler and therefore preferable representations for specialized types of waffle stiffening are given directly in terms of orthotropic-plate properties in references 73, 97, and 98. Available references present theory only; no comprehensive design charts can be recommended.

4.2.3 Classical Methods

Classical formulations and supporting tabular data have been substantiated for many cases of plate analysis and are available in the published literature. These data should be used in determining the buckling and crippling strengths of structural plates whenever possible, especially for the conventional forms of plate construction (unstiffened,

stringer stiffened, and sandwich) which have a rectangular planform and are subjected to uniformly distributed loads.

Plates with special features should also be analyzed by classical methods if such features are amenable to representation in simple functional form, as described in Section 2.1. Examples of classical methods applied to thermal-stress, residual-stress, initial-imperfection, and tapered-thickness analyses are given in references 99, 91, 40, and 100, respectively. Time-shared, remote-access computer facilities with substantial capacity make these types of analyses available to most designers. Care should be exercised, however, in determining the number of terms used in the functional representations of the problem (refs. 4 to 6). If the circumstances of a problem are too complex for adequate functional representation, the numerical methods described in Sections 2.2 and 4.2.2 should be employed.

Recommendations for the use of classical methods in determining the buckling and crippling strengths of unstiffened (isotropic), fiber-reinforced composite, corrugated, waffle-stiffened, stringer-stiffened, and sandwich plates follow. Since there is an immense body of published data for these forms of construction, emphasis in the following discussion is placed on references to sources for these data, rather than on the presentation of extensive formulas and detailed design guides.

4.2.3.1 Unstiffened (Isotropic)

As the basis for calculating the buckling strength of unstiffened isotropic plates of conventional materials and for evaluating local failure modes of other forms of plate construction, the following characteristic formula for critical stress should be used:

$$\sigma_{cr} = \eta_c \eta_p \frac{kE\left(\frac{t}{b}\right)^2}{12(1-\mu^2)} = \eta_c \eta_p \, k\frac{D}{tb^2} \qquad (1)$$

where σ_{cr} is the value of the reference stress (i.e., the stress at a point selected to characterize the overall distribution of stress in the middle surface of the plate); E and μ are the elastic modulus and Poisson's ratio, respectively; η_c and η_p are the reduction factors for cladding and plasticity, respectively; k is the buckling coefficient; and D is the plate's flexural stiffness per unit width. The geometry of the plate is in part represented by the plate thickness, t, and a characteristic width dimension, b. Under uniaxial compression, b is the width of the plate.

Reference 7 should be used for information on cladding-reduction factors. Plasticity-reduction factors should be computed from deformation theory (refs. 24 and 26) or should be obtained from charts derived from this theory (ref. 101).

The buckling coefficient is a function of the loading distribution and of the plate geometry and boundary conditions. Equation (1) applies directly to conditions where a uniform uniaxial or constant shear stress prevails [although the equation can be employed in more complex situations by defining σ_{cr} as a reference value, as in eq. (1)]. Buckling coefficients for a wide range of nonuniform uniaxial-load conditions and support arrangements can be found in standard texts (refs. 4 to 6).

The range of possible combinations in multiaxial-stress states is too large to permit efficient data representations through the device of a single reference value. Therefore, the stress-ratio concept in the form of interaction formulas should be used for multiaxial-stress conditions. A wide range of interaction formulas covering combined inplane-bending stress and shear is presented in references 7, 8, and 92.

Buckling strengths of uniaxially compressed, unstiffened isotropic plates with curvature in the direction normal to the loading should be determined by using the empirical method originally given in reference 51, and more recently described in reference 70. This approach is based on a comparison of a curvature parameter, Z, with a parameter describing the severity of initial imperfections, γ, where

$$Z = \frac{b^2}{rt}\sqrt{1-\mu^2} \tag{2}$$

and

$$\gamma = \frac{\pi^2 k}{2\sqrt{3}}\sqrt{\frac{1}{1+6\beta\frac{r}{t}}} \tag{3}$$

in which b is the characteristic plate dimension, r is the radius of curvature, k is the buckling coefficient for the corresponding flat plate, and β defines the amplitude of the initial imperfections. For $Z < \gamma$, the curved plate is treated as a flat plate with modified buckling coefficient, while for $Z > \gamma$, the governing stability equation is of the general form of the cylinder-stability equation. (See ref. 1 for discussion of the latter.) Estimates of the parameter β and the detailed forms of the governing equations for this method may be obtained from reference 70, but preferably, the parameter β should be determined by testing plates of the same fabricational quality as the plates being analyzed.

Initial buckling due to thermal gradients should be determined by procedures described in references 46 and 48. These procedures are directed mainly toward the problem of instability under the condition of a nonuniform distribution of middle-surface stress caused by thermal gradients. References 44 and 45 should be consulted for sources of

the limited available design data based on these procedures. Since available classical procedures and design data are largely applicable only to one particular circumstance, it should be expected that numerical approaches will generally be necessary to determine temperature changes that cause buckling.

In general, if temperature change causes no greater than a 10-percent difference in material-property values within a plate, then the approximation of material-property values at average temperature should be satisfactory in determining thermal buckling. However, this approach is not likely to reduce effort significantly when numerical methods are applied. If the temperature gradient is highly localized, use of average-temperature property values is also permissible even when property values vary more than 10 percent.

If the postbuckling stiffness of a plate must be known for analyses of the overall structure of which the plate forms a part, it is recommended that the design charts presented in Appendix A of reference 35 be used for biaxially applied stress states. Reference 35 also describes a computer program capable of performing such stiffness determinations. An empirically based formula (ref. 102) is recommended as a simple alternative for determining postbuckling stiffness in the special case of uniaxial compression. For determining the shear stiffness of a plate buckled in axial compression, reference 36 should be employed. For plates buckled under pure shear loadings, the formulations derived in reference 36 or 38 should be used; both of these references contain simple design formulas and graphical data.

Plastic buckling and crippling should be regarded as synonymous. The crippling strength of the flat isotropic plate that buckles elastically may be substantial. In theory, this strength is a complicated function of the support conditions, the loading distribution, and the material's mechanical properties. Crippling strength can be determined by completely theoretical means using digital computers. For uniaxial compression, however, simplified expressions based upon effective-width concepts should be adequate for design purposes. Effective-width formulas for plates whose middle-surface displacements in the direction normal to the unloaded edges are constrained should be obtained from references 8 and 10. The empirical effective-width formula of reference 103 is recommended for plates with edges free to move in the plane of the plate. This formula has been substantiated by recent theoretical studies (ref. 39) and extensive experimental data.

No theoretical methods to determine crippling in the presence of thermal strain can be recommended. A simple approximate method for determining the onset of permanent buckling due to thermal- and compressive-load stresses developed in reference 60 can be recommended although this procedure is substantiated by only limited test data.

Reference 43 presents a method which is recommended for determining the crippling strength of a plate from data available for a plate of a different material. The correlation of the crippling strength of plates of different materials depends upon a material parameter that is a function of the compressive-yield strength and the secant modulus of elasticity. Recent test data (ref. 104) suggest that this method may be unsatisfactory for application to complex titanium-plate structures.

For determining the crippling strength of singly curved plates under complex loads, flat-plate theory should be used in conformance with the conjecture of reference 55. For doubly curved plates, the absence of data also makes it necessary to use flat-plate theory to determine crippling strength.

4.2.3.2 Fiber-Reinforced Composite

As noted in Section 2.2.6, orthotropic models are generally used in the determination of elastic-buckling stresses of fiber-reinforced composite plates. Reference 13 is recommended as an authoritative source of theory, background information, and formulas for stability analysis of orthotropic plates. Reference 14 should also be consulted as a means of maintaining awareness of current work. Since the equations and associated design data for fiber-composite plates cover only a limited range of the governing parameters, it may be necessary to employ numerical methods or series-solution procedures with classical methods.

The reduced bending-stiffness concept and other simplifications intended to make fiber-composite-plate analysis amenable to simple orthotropic-plate theory are usually acceptable in design practice within the limits discussed in references 85 and 86. These simplifications, however, err on the unconservative side and should be checked by an examination of simple test cases.

Because of delamination and the absence of substantiated analysis procedures, either plastic buckling and postbuckling situations should be avoided in the design of fiber-reinforced plates or expected behavior in these situations should be verified by test. Limited experimental evidence (ref. 89) shows that when plates have a sufficient number of plies to be isotropic in behavior, the effective-width concept for determining crippling strength is valid. Even in such cases, however, delamination and other local failure mechanisms can be expected to limit the plate strength to that of initial buckling.

For laminated-composite plates, it is considerably more difficult to check for yielding under direct compression before buckling than it is for isotropic plates. The yield condition should be determined on the basis of the yield strength of each orthotropic lamina in reference to the lamina's principal axes since the yield strength is established

experimentally with respect to these axes. Depending upon the nature of the plate structure, the procedures described in references 87 and 88 are acceptable.

Uncertainties in quality control and tolerances in fabrication, along with the lack of understanding of critical failure mechanisms at the level of the individual fiber, should be considered as potential problem sources in the practical analysis of the elastic stability and buckling strength of fiber-reinforced composite plates.

4.2.3.3 Corrugated

Orthotropic-plate theories which account for shear deformation in the plane perpendicular to the corrugations are preferred for initial buckling analyses of corrugated plates. Shear stiffnesses for use in such analyses should be obtained from references 60 and 61, or computed from the types of procedures they describe. Reference 61 should also be consulted for appropriate stiffness parameters for plates with intermittent attachments at the corrugation ends.

If shear-stiffness effects can be shown to be negligible by analysis or test, solutions for the initial buckling of corrugated plates should be taken from references 4, 13, 57, and 58. Available data and experimental correlations are insufficient to permit recommendation of specific procedures for determining postbuckling strength, local buckling, and inelastic buckling of corrugated plates.

4.2.3.4 Waffle Stiffened

Calculations for determining the instability characteristics of waffle-stiffened plates are based on orthotropic-plate theory so that the foregoing comments regarding fiber-reinforced composite plates are applicable here as well. Local instability in the form of independent buckling of the ribs and sheet plates is a possible failure mode for waffle plates (ref. 105). Although no experimental studies have dealt specifically with plastic buckling and postbuckling strengths of waffle plates, it appears reasonable to use the procedures recommended for unstiffened and stringer-stiffened plates to determine these strengths.

4.2.3.5 Stringer Stiffened

Stringer-stiffened plates are subject to failure as (1) long plates in Euler (column) buckling; (2) short plates in a variety of local buckling or crippling modes; and (3) intermediate-length plates in a combined crippling and column mode. Because of the many possible failure modes and the wide range of stiffener shapes and spacings, it is not feasible to recommend specific formulations for this type of construction.

Reference 10 should be consulted for details on available computational procedures substantiated by much experimental data accumulated over many years; reference 7 is especially recommended as a source of graphical design data. References 4, 5, and 11 should be consulted concerning other failure modes (e.g., the torsional rolling mode of open-section stiffener buckling).

In addition to the modes of instability cited, attention should be given to instability modes associated with skin-to-stringer attachments. Interrivet buckling should be examined with the use of the procedures and data given in references 7, 10, and 106. Another attachment-failure mode that should be checked for is forced crippling. This occurs when a thick skin buckles and the stiffener is forced to follow the skin deformations. Formulations for analysis of this condition are summarized in reference 10.

Advances made after the publication of references 7 and 10 in defining local buckling modes of stringer-stiffened plates are described in reference 64; advances in the treatment of inelastic behavior and shear loadings, in references 24 and 107; and advances in the representation of stiffener eccentricity, in reference 65. Reference 65 is particularly helpful for identifying some of the inadequacies of orthotropic-plate theory as an approximate representation of overall buckling.

In the stability analysis of stringer-stiffened plates, attention should also be given to emerging theoretical procedures founded in classical analysis concepts but which require digital-computer capabilities for application (ref. 66). These procedures employ a general representation of structural geometry and behavior, and should eliminate the need for independent study of numerous estimated buckling modes.

Zee-shaped stiffeners are used more often than other shaped stiffeners on curved plates. The buckling of singly curved, Zee-section-stiffened plates should be treated by application of the procedure given in reference 70, (discussed in the preceding sections in connection with isotropic unstiffened plates). Reference 70 also contains an extension of available solutions for the buckling coefficient of flat Zee-stiffened plates. For stringer-stiffened plates, the recommended procedure again involves a comparison of curvature- and imperfection-sensitivity parameters, Z and γ, respectively.

4.2.3.6 Sandwich

Sandwich plates should be checked for three possible modes of instability failure: (1) general failure, with core and facings acting together; (2) local failure in the form of dimpling, wrinkling, or delamination of a face, or shear buckling of the core; and (3) local buckling of a plate element of the core, as in truss-core or corrugated-core sandwich construction.

Reference 84 should be consulted for formulas used in analyzing both local and general instability. This document critically reviews, correlates, and summarizes recommended formulas for nearly all aspects of sandwich-plate analysis for which solutions have been published. Reference 108 should be consulted for formulas used in analyzing sandwich plates with fiber-composite face sheets. Reference 93 should be used as an authoritative source of sandwich-plate design practice and data, but it does not discuss underlying theory; it is oriented toward the selection of plate proportions to meet specified buckling strengths for highly regular conditions. References 82 and 83 should be consulted for a detailed discussion of theoretical approaches to general and local instability.

Solutions for the thermal buckling of sandwich plates are given in graphical form in reference 109. Because no data can be recommended for determining the influence of attachments on sandwich-plate instability, test data should be obtained for each case.

From a design standpoint, experimental evidence (chap. 6 of ref. 81 and Sec. 9 of ref. 83) indicates that it is acceptable to apply the plasticity-reduction factor for unstiffened plates to determine both general and local instability phenomena in sandwich plates. Reference 84 should be consulted for those plasticity-reduction-factor formulas appropriate to general conditions and those appropriate to local conditions. An incomplete understanding of local failure mechanisms and core-to-face bond strength, however, requires that no allowance be made in design for postbuckling strength.

Formulas for local instability are not reliable for the complete range of parameters in practical sandwich construction. For heavy cores (i.e., where the ratio of core density to face-sheet density exceeds approximately 0.03), the approximate equations for local instability should be adequate. Heavy cores do not impose a significant weight penalty upon the total plate design and are therefore recommended as a means of effecting large margins of safety against failure in local buckling modes.

For corrugated-core sandwich plates, references 110, 111, and 112 should be consulted for design data and formulations for overall buckling, for local buckling and postbuckling strength, and for plastic buckling, respectively.

4.2.4 Numerical Methods

Numerical methods are recommended for analyses that are too complex for the more economical classical approaches. Advantage should be taken of the ability of numerical methods to cope with flat or curved plates that (1) are formed in irregular shapes and varying proportions; (2) have stiffeners, reinforced cutouts, and unusual support conditions; and/or (3) are subjected to nonuniformly distributed loads and temperatures.

Such methods are also effective in accounting for initial imperfections, residual stresses inelastic- and anisotropic-material properties, and the influence of temperature variations on mechanical and physical properties of materials.

The accuracy of a numerical solution can be compromised by data-preparation errors, the format of the mathematical model, and the arithmetical procedures of the computational system. Reference 113 should be consulted for safeguards against these sources of error.

When experimental or analytical verification is lacking, numerical solutions should be substantiated by one of two alternative procedures. In one, solutions should be obtained from analytical models that are successively more refined and the calculated critical loads of the successive analyses compared to determine convergence trends. Rules for extrapolation to estimate the convergent solution from these comparisons have been developed for finite-difference analysis (ref. 15) and can be applied to finite-element calculations, as well.

The second substantiation procedure uses two mathematical models, both constructed on the same level of grid refinement. Upper and lower bounds for the convergent solution are established by constructing one of the two models to ensure that the calculated buckling load will be higher than the convergent solution, while the other analytical model is constructed to yield a buckling load lower than the convergent solution. The upper-bound model, formulated in conformity with the principle of stationary potential energy, requires the definition of assumed displacements in an appropriate form. Lower-bound evaluation procedures have not yet been explored in finite-element buckling analysis, although their role in analyses for stable conditions is well explored. References 114 and 115 should be consulted for theoretical and practical aspects of evaluating lower bounds for convergent solutions. When a converged solution has been obtained, the buckling-mode shape should be computed to assist in identification of the critical element for stability.

System-reliability requirements, fabricational tolerances, and the degree of uncertainty associated with analysis-input data should be assessed in determining the accuracies required in solutions. In general, however, calculated results should correspond to the convergent solution for the mathematical model within 5 percent.

Numerical methods are not well suited for incorporating simplifications (such as the effective-width concept) which are used in elementary analytical theory for determining postbuckling strength. On the other hand, numerical methods can be directly applied to more rigorous and realistic exercises in computing postbuckling phenomena, particularly in situations with unusual stress-strain laws, and in computing

combined inelastic- and finite-displacement behavior. Reference 116 and 117 should be examined for descriptions of the procedures to follow in these computations.

It is important to note that current consideration of the finite-element method for evaluating plate buckling may require extension of existing element formulations for plate flexure. A great many alternative representations of these relationships have been published; these are summarized in references 118 and 119. Any valid mathematical model is an acceptable choice for extension of the element formulation to the elastic-instability case, whether it satisfies either equilibrium or compatibility conditions, or neither, throughout the element and system. It is desirable to retain the functional representations of element behavior as originally used in the basic flexural representation in the development of the additional terms required for instability analysis. Numerical-method formulations should be validated by analyses for which solutions are known.

4.3 Tests

4.3.1 Materials

Compression tests or shear tests should be performed, as appropriate, to establish mechanical-property values for materials not covered by tabulated data or where special conditions are anticipated. For example, reference 120 demonstrates that the reduction in the proportional-limit stress due to fatigue will reduce buckling strength. A simulation of service history, followed by performance of the standard material-properties tests, may therefore be necessary for a structure designed for long life. Tests should conform to the standards given in references 121 and 122.

4.3.2 Plate Structures

Experimental data on the buckling and crippling strengths of structural plates are especially needed for designs with irregular geometry, cutouts, and combined thermal and mechanical loads.

Specimens should possess the initial-imperfection and residual-stress qualities of the actual structure, and a sufficient number of specimens should be tested to provide a valid statistical sample. Whenever possible, tests should be conducted with full-scale specimens and with realistic simulation of the continuous adjacent structure. When single plates are tested in isolation, it is difficult to simulate any adjacent structure conditions other than the extremes of fully fixed, free, or simple support. References 40, 123, and 124 should be consulted for descriptions of plate-buckling test-support conditions. Care should be exercised to interpret test data properly for flat-end

tests, for which the assumption of full or nearly complete fixity is often made but not always realized. Also, in tests of postbuckling behavior and crippling strength, particular attention should be given to the support conditions along the unloaded edges due to their major influence on behavior.

Care should be taken that test loads are introduced into the plate in the same manner as anticipated for the real structure, and that the elastic response of the test fixture does not influence the buckling and crippling loads. Thermal loads can be imposed by means of radiant- or induction-heating facilities (ref. 125). Thermal testing usually involves the imposition of a specified variation of temperature throughout the panel, accompanied by a gradual increase of load until buckling or crippling occurs. It is often impractical to maintain the specified temperature by control of the thermal-input devices and the laboratory environment alone. Coolant tubes, attached to or incorporated in the test specimens, have been successfully used in providing more practical thermal control (refs. 126 and 127).

Because of its initial imperfections, a plate being tested will seldom experience buckling at a sharply defined load level. Various methods for identifying the buckling load from measured test data are widely used (fig. 5). The method of Southwell (refs. 128 to 130) interprets the critical stress as the slope of the straight line passing through points which give the maximum normal displacement (δ) versus the ratio of this displacement and the applied stress (δ/σ) (fig. 5a). Particular attention should be given to the interpretation of these points; the measured deflection must be small enough so that the small-deflection theory remains applicable and yet large enough so that the amplitude of the buckling mode, rather than the initial displacement, predominates.

Another approach to the identification of buckling loads is the "top-of-the-knee" method (refs. 89 and 123), illustrated in figure 5b. According to this method, the buckling load is essentially the load corresponding to the top of the knee of a curve of stress versus the lateral deflection. Reference 123 also describes the "strain-reversal" method, in which the buckling load is taken as the load at which the strain at a point on the crest of the buckle stops increasing and starts decreasing (fig. 5c).

In another approach for interpreting test data to identify the buckling load (ref. 123), the average stress on the plate is plotted against the unit-compressive strain (the total shortening of the plate divided by the plate length), as illustrated in figure 5d. The buckling stress is then considered as the point where a sharp reduction in slope appears.

Still another method (ref. 131) is based upon the interpretation of a plot of the unit shortening versus the square of the maximum deflection of the plate.

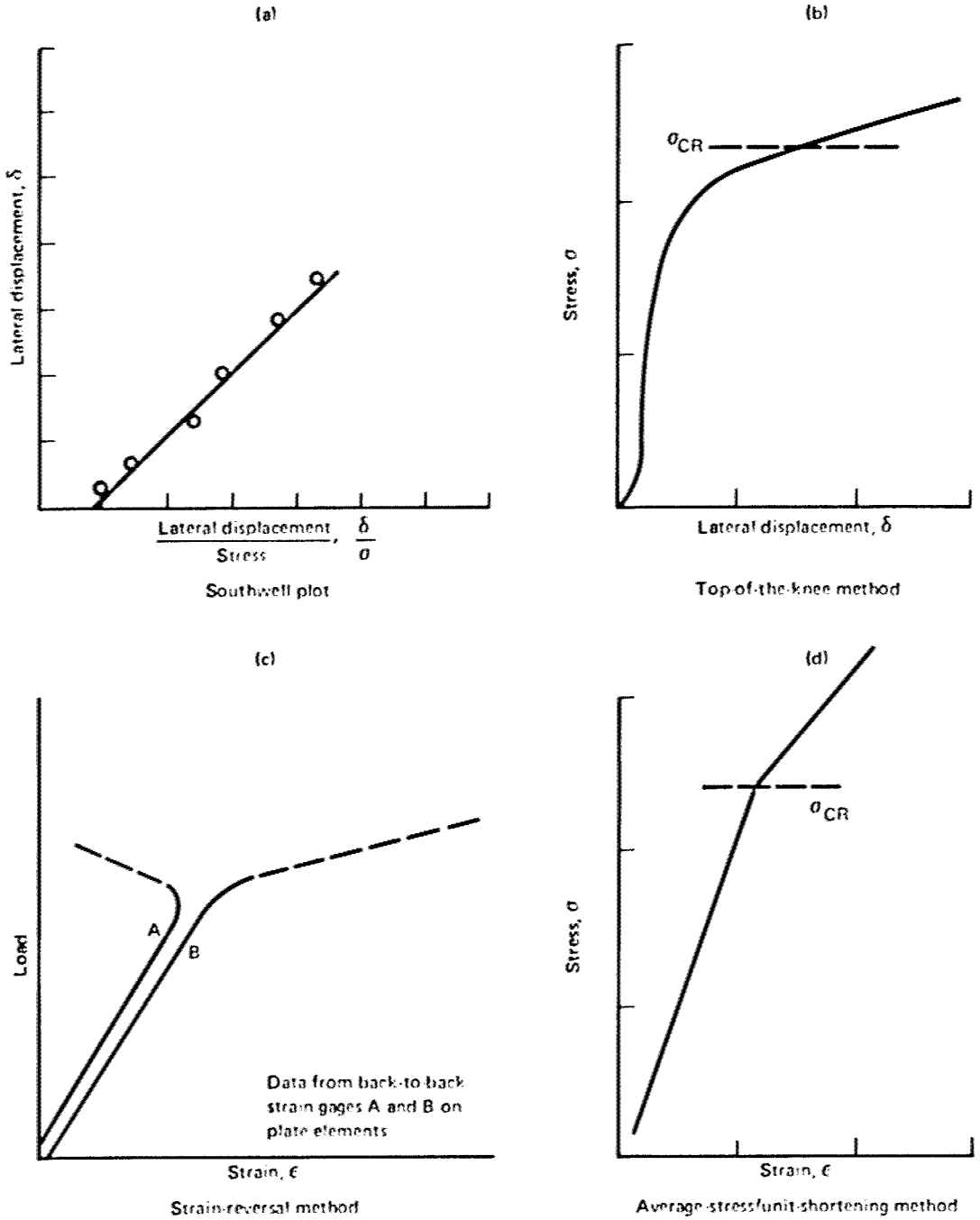


Figure 5. – Methods for identifying buckling stress from experimental data.

Each of the foregoing procedures has been employed with success in both plastic- and elastic-buckling load evaluations (refs. 123, 130, and 131). It is recommended that the strain-reversal method be used in conjunction with at least one of the other methods described above. The tests should reveal both the buckling and crippling strengths of the plate structure.

Test instrumentation should include strain gages and deflection transducers. Since instability represents a transition from compressive to flexural behavior and the interpretation of test data can depend upon assessment of net compressive strain, care should be taken to locate an adequate number of strain gages on both faces of the plate structure (back to back) to ensure identification of the onset of buckling.

References 84, 124, and 132 should be consulted for modern practice in the instability testing of special forms of plate construction, such as fiber-reinforced composite, sandwich, and stringer-stiffened plates.

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